Workshop on Algebraic Combinatorics 2015

Tilburg University, The Netherlands

17 & 18 June 2015

`http://workshoponalgebraiccombinatorics2015.weebly.com/`

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We welcome you to Tilburg for a workshop on algebraic combinatorics. This contains important practical and scientific information about the meeting. To conserve paper, we provide at the meeting printed copies of only the programme. This document has links, via hyperref, to other parts of the document and the web.

Practical matters

The entire workshop takes place in the Dante building on the campus of Tilburg University. Talks on June 17 are in the room DZ004, and talks on June 18 are in the room DZ003, both located on the ground floor not far from the building entrance.

The campus is about 3 kilometres west of the city centre. To get there by bus (stop "Tilburg Universiteit"), use bus number 4 (direction "Reeshoof") from Tilburg central station. You can also reach Tilburg University by train from Tilburg central station (stop "Tilburg Universiteit"). It can be a 10 minute journey if you have a bicycle. There is eduroam. Remember to configure it at your home institution beforehand.

During the meeting, we provide caffeine, lunch and other refreshments at the right moments. There is a lunch included on Thursday.

Tilburg can be reached by a 90-minute train journey from Schiphol airport (with a quick change at 's-Hertogenbosch). Also from Schiphol, there is a 75-minute trip by train via Breda. Other airports nearby are Eindhoven and Rotterdam/Den Haag. Consult `http://9292.nl/` to check the overall route. For train routes you can also check `http://www.ns.nl/en/travellers/home`. Note that limited free internet access is usually provided at Schiphol airport, major train stations, and on Intercity trains. To plan meals for the evenings, search on the website `http://en.iens.nl/`.

Programme

Wednesday, 17 June 2015

13:30-14:00	Coffee/tea
14:00-14:40	**Miquel Angel Fiol** - *Quotient-polynomial graphs*
14:40-14:50	Break
14:50-15:10	Margarida Mitjana - *The Inverse Matrix of some Circulant and Symmetric Matrices*
15:10-15:30	Jae-Ho Lee - *Nonsymmetric Askey-Wilson polynomials and Q-polynomial distance-regular graphs*
15:30-15:50	Ángeles Carmona - *Equilibrium measure of regular-layered networks*
15:50-16:20	Coffee/tea break
16:20-17:00	**Aart Blokhuis** - *The structure of small and large super-Vandermonde sets*

Thursday, 18 June 2015

9:30-9:50	Simeon Ball - *Arcs and inclusion matrices*
9:50-10:10	Silvia Gago - *Green matrices of graphs with new independent vertices*
10:10-10:30	Maarten De Boeck - *A linear set view on KM-arcs in* $PG(2, q)$
10:30-11:00	Coffee/tea break (conference photo first)
11:00-11:40	**Oriol Serra** - *Dimensional versions of sumset inequalities*
11:40-11:50	Break
11:50-12:10	Bernardo Rodrigues - *Codes from incidence matrices of graphs*
12:10-12:30	Gary Greaves - *Graphs with three eigenvalues*

14:00-14:40	**Monique Laurent** - *Matrix completion and geometric graph realizations*
14:40-14:50	Break
14:50-15:10	Sonia Pérez - *A family of Abelian Cayley digraphs with asymptotically large order*
15:10-15:30	Ferdinand Ihringer - *Erdős-Ko-Rado Sets and Semidefinite Programming in Hermitian Polar Spaces*
15:30-15:50	Andrés Encinas - *Perturbations of discrete elliptic operators*
15:50-16:20	Coffee/tea break
16:20-17:00	**Sebastian Cioabă** - *Large regular graphs with given valency and second eigenvalue*

Invited lectures

Aart Blokhuis (Technische Universiteit Eindhoven, The Netherlands)
The structure of small and large super-Vandermonde sets

A *super-Vandermonde set* in $GF(q)$ is a set T of size $1 < t < q$ such that $\pi_k(T) = \sum_{\tau\in T}\tau^k = 0$ for $1 \le k \le t-1$. It is 'easy to see' that $0 \notin T$ and that $\pi = \pi_t(T) \neq 0$ (in particular $p \not| t$). Let $T = \{y_1, \ldots, y_t\}$, then being super-Vandermonde is equivalent to

$$\prod_{i=1}^{t}(Y - y_i^{-1}) = Y^t + g(Y),$$

with g a p-th power.

Part of the thesis of Marcella Takáts is about small and large super-Vandermonde sets. Here small means $t \le p$, and small super-Vandermonde sets are (essentially, that is cosets of) multiplicative subgroups of $GF(q)^*$: In this case the polynomial g is constant, so

$$\prod_{i=1}^{t}(Y - y_i^{-1}) = Y^t - c,$$

where $t \,|\, q-1$ and c is a t-th power. Large means $t \ge q/p$ and again we get (essentially) multiplicative subgroups.
The two bounds are natural, in my talk I will classify the (infinitely many) super-Vandermonde sets of (small) size $p+1$ and (large) size $q/p-1$.

Sebastian Cioabă (University of Delaware, USA)
Large regular graphs with given valency and second eigenvalue

From Alon and Boppana, and Serre, we know that for any given integer $k \ge 3$ and real number $\lambda < 2\sqrt{k-1}$, there are finitely many k-regular graphs whose second largest eigenvalue is at most λ. In this talk, we will focus on determining the largest number of vertices of such graphs.
This is based on joint work with Jack Koolen, Hiroshi Nozaki and Jason Vermette.

Miquel Àngel Fiol (Polytechnic University of Catalonia, Spain)
Quotient-polynomial graphs

As a generalization of orbit-polynomial and distance-regular graphs, we introduce the concept of a quotient-polynomial graph. In these graphs every vertex u induces the same regular partition around u, where all vertices of each cell are equidistant from u. Some properties and characterizations of such graphs are studied. For instance, all quotient-polynomial graphs are walk-regular and distance-polynomial. As a by product, we give a characterization of those vertex-transitive graph which are distance-regular. Also, we show that every quotient-polynomial graph generates a (symmetric) association scheme.

Monique Laurent (CWI & Tilburg University, The Netherlands)
Matrix completion and geometric graph realizations

We consider the problem of completing a partially specified matrix to a positive semidefinite matrix, with special focus on questions related to the smallest possible rank of such completion. We present complexity results and structural characterizations of the graph of specified entries for the existence of small rank completions, as well as links to Euclidean graph realizations in distance geometry and to some Colin de Verdière type graph parameters.

Oriol Serra (Polytechnic University of Catalonia, Spain)
Dimensional versions of sumset inequalities

Inverse problems in additive theory aim to provide structural results of sets in an additive group which have a small sumset. Motivated by a problem on difference sets, Hou, Cheng and Xiang obtained a linear analogue of one the central results in the area, the theorem of Kneser, in which cardinalities of sets in an abelian group are substituted by dimensions of subspaces over a field. One of the nice features of this dimension version is that it gives the classical one as a Corollary.
We will discuss a dimension version of one of the basic inverse theorems in additive combinatorics, the theorem of Vosper. The proof combines ideas from additive combinatorics, by extending the so-called isoperimetric method to the dimension setting, and also use results on quadratic forms and what has become known as the linear programming method in the theory of error-correcting codes. On the way the nonexistence of maximum distance separating codes in a space of bilinear forms with respect to a natural metric is obtained, a result which has interest in itself.
This is joint work with Christine Bachoc and Gilles Zemor.

Contributed talks

Simeon Ball (Polytechnic University of Catalonia, Spain)
Arcs and inclusion matrices

An arc S in the projective space $\mathrm{PG}_{k-1}(\mathbb{F}_q)$ is a set of a points with the property that every k points of S span the whole space. If $k \geqslant q$ then it is relatively easy to show that $|S| \leqslant k+1$, the bound is attainable and sets attaining the bound are easily classified. The MDS conjecture states that if $k \leqslant q$ then $|S| \leqslant q+1$, unless q is even and $k=3$ or $q-2$ in which case $|S| = q+2$ is best possible. The MDS conjecture is true for q prime, the proof of which combines Segre's lemma of tangents with interpolation. For q non-prime, there are proofs of the MDS conjecture for small $k < c\sqrt{pq}$, where q is a power of a prime p. All but one of these partial proofs use the existence of an algebraic variety of small degree whose zeros contain all the points dual to hyperplanes containing exactly $k-1$ points of S. The existence of this algebraic variety also follows from Segre's lemma of tangents. The other partial proof of the MDS conjecture (for $k \leqslant 2p-2$) uses again Segre's lemma of tangents and interpolation. The aim of this talk is to show how the existence of the algebraic hypersurface follows from the interpolation approach, how inclusion matrices can be used to simplify previous work, and what we can (hope to) prove using inclusion matrices.

Maarten De Boeck (Ghent University, Belgium)
A linear set view on KM-arcs in PG$(2,q)$

A *KM-arc of type* t in $\mathrm{PG}(2,q)$ is a set of $q+t$ points in the Desarguesian projective plane $\mathrm{PG}(2,q)$ such that any line meets it in 0, 2 or t points, with $2 \le t < q$. These sets are named after Korchmáros and Mazzocca who introduced and investigated these point sets in [3]. If a KM-arc of type t in $\mathrm{PG}(2,q)$ exists, then q is even and t is a divisor of q. Further theoretical results were obtained in [2]. A KM-arc $\mathcal{A}$ is called a *translation* KM-arc if it is a translation set, i.e. there is a line ℓ (called the translation line) such that the group of all translations fixing ℓ acts transitively on the points of $\mathcal{A} \setminus \ell$.

In [3] Korchmáros and Mazzocca constructed KM-arcs of type 2^i in $\mathrm{PG}(2,2^h)$ for all $i > 0$ such that $h-i$ is a divisor of h. Their construction is algebraic and is based on the trace function. In [2] Gács and Weiner gave a geometrical description of the KM-arcs described in [3] and constructed (in a geometrical way) a family of KM-arcs of type 2^i in $\mathrm{PG}(2,2^h)$ with $h-i+1$ a divisor of h. Vandendriessche constructed in [4] a family of KM-arcs of type $\frac{q}{4}$ in $\mathrm{PG}(2,q)$.

In this talk, based on [1], we discuss two results regarding KM-arcs. First we present a characterisation of translation KM-arcs based on a specific class of linear sets, called *i-clubs*. We give a more algebraic presentation of the KM-arcs described in [2] and reinterpret the translation KM-arcs of [2, 3, 4] in view of these i-clubs. In particular we show that all the examples of [4] are translation KM-arcs. Secondly we describe a family of KM-arcs of type $\frac{q}{4}$ in $\mathrm{PG}(2,q)$, generalising the family found in [4]. It includes both translation and non-translation KM-arcs.

References

[1] M. De Boeck and G. Van de Voorde, A linear set view on KM-arcs. *Submitted* to *J. Algebraic Combin.*, (2014), 31 pp.

[2] A. Gács and Zs. Weiner, On $(q+t)$-arcs of type $(0,2,t)$. *Des. Codes Cryptogr.*, **29 (1-3)** (2003), 131–139.

[3] G. Korchmáros and F. Mazzocca, On $(q+t)$-arcs of type $(0,2,t)$ in a desarguesian plane of order q, *Math. Proc. Cambridge Philos. Soc.*, **108 (3)** (1990), 445–459.

[4] P. Vandendriessche, Codes of Desarguesian projective planes of even order, projective triads and $(q+t,t)$-arcs of type $(0,2,t)$, *Finite Fields Appl.*, **17 (6)** (2011), 521–531.

Ángeles Carmona (Polytechnic University of Catalonia, Spain)
Equilibrium measure of regular-layered networks

We consider here the discrete analogue of Serrin's problem: if the equilibrium measure of a network with boundary satis?es that its normal derivative is constant, what can be said about the structure of the network and the symmetry of the equilibrium measure? In the original Serrin's problem, the conclusion is that the domain is a ball and the solution is radial. To study the discrete Serrin's problem, we ?rst introduce the notion of radial function and then prove a generalization of the minimum principle, which is one of the main tools in the continuous case. Moreover, we obtain similar results to those of the continuous case for some families of networks with a ball-like structure, which include spider networks with radial conductances, distance-regular graphs or, more generally, regular layered networks.

Andrés Encinas (Polytechnic University of Catalonia, Spain)
Perturbations of discrete elliptic operators

A self-adjoint operator K on a finite network is called elliptic if it is positive semi-definite and its lowest eigenvalue is simple. Examples of elliptic operators are the so-called Schrödinger operators on finite connected networks, as well as the signless Laplacian of connected bipartite networks. Any elliptic operator, determines an automorphism on the orthogonal complement of the subspace of the eigenfunctions corresponding to the lowest eigenvalue. Its inverse is is called orthogonal Green operator of K. We aim here at studying the effect of a perturbation of K on its orthogonal Green operator. The perturbation here considered is performed by adding a self-adjoint and positive semi-definite operator to K. As particular cases we consider the effect of changing the conductances on semi-definite Schrödinger operators on finite connected networks and on the signless Laplacian of connected bipartite networks. The expression obtained for the perturbed network is explicitly given in terms of the orthogonal Green function of the original network.

Silvia Gago (Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona, Consorci Escola Industrial de Barcelona, Departament Matemàtica Aplicada 3, Polytechnic University of Catalonia, Spain)
Green matrices of graphs with new independent vertices

The computation of the Green matrix of a weighted graph is a subject of great interest because of its wide range of applications. In particular, when the potential of the Schrödinger matrix is zero, the inverse of the Laplacian matrix is obtained. The effective resistances and Kirchhoff index of the weighted graph can be also obtained in terms of it. In previous works, the Green matrix of weighted graphs obtained with some operations has been studied, as the Green matrix of join graphs, generalized linear polyominos, cluster graphs, etc. as well as the effect of the perturbation of the conductance of some edges of the graph.
In this work we aim to study the effect on the Green matrix of the graph when both we add several new independent vertices in terms of the Green matrix of the original network. As an example of application, we obtain the Green matrix of a stairs graph.
This is based on joint work with A. Carmona, A.M. Encinas and M. Mitjana.

Gary Greaves (Tohoku University, Japan)
Graphs with three eigenvalues

I will present some recent results on graphs having precisely three eigenvalues. In particular, I will present results about the case when such graphs have only two different valencies and the case when the second largest eigenvalue is at most 1.

Jae-Ho Lee (Tohoku University, Japan)
Nonsymmetric Askey-Wilson polynomials and Q-polynomial distance-regular graphs

Nonsymmetric Askey-Wilson polynomials were defined by Sahi in 1999. Roughly speaking, they are the eigenfunctions of the Cherednik-Dunkl operator and form a linear basis of the space of the Laurent polynomials in one variable. In my thesis (2013), we found a relationship between Q-polynomial distance-regular graphs and a double affine Hekce algebra of rank one. In this talk, using this relationship we will define the nonsymmetric Askey-Wilson polynomials in a different way and discuss how these polynomials are related with Sahi's nonsymmetric Askey-Wilson polynomials.

Ferdinand Ihringer (Justus-Liebig-Universität Gießen, Germany)
Erdős-Ko-Rado Sets and Semidefinite Programming in Hermitian Polar Spaces

Let V be an 6-dimensional vector space over a finite field K of order q^2 and consider the sesquilinear form $s : V \times V \longrightarrow K$ defined by

$$s(x,y) = x_0 y_0^q + x_1 y_1^q + x_2 y_2^q + x_3 y_3^q + x_4 y_4^q + x_6 y_6^q.$$

The totally isotropic subspaces of V with respect to s define an incidence geometry $\mathsf{H}(5,q^2)$, which consists of points, lines and planes. If we say that two planes of $\mathsf{H}(5,q^2)$ are adjacent if and only if they meet in a line, then this defines the dual polar graph $\mathsf{U}(6,q)$.

An *Erdős-Ko-Rado set* Y of $\mathsf{H}(5,q^2)$ is a set of planes, which pairwise meet non-trivially. A non-trivial problem is to provide a tight upper bound on Y. The analog problem for $\mathsf{H}(2d-1,q^2)$ (with totally isotropic subspaces of dimension d instead of planes) is open for $d > 3$ odd. For $d = 3$ the only previously known proof of the tight bound is geometric and does not seem to generalize to $d > 3$ (see [1]).

We define a *coherent configuration* on $\mathsf{H}(5,q^2)$, which is closely related to the *Terwilliger algebra* of the graph $\mathsf{U}(6,q)$. We apply Hobart's semidefinite programming bound (see [2, 3]) to obtain a first algebraic proof of an EKR theorem of $\mathsf{H}(5,q^2)$. By the time of the talk it might be clear if this technique extends to the first open case, $\mathsf{H}(9,4)$. Furthermore, we mention some other applications of semidefinite programming in Hermitian polar spaces.

References

[1] V. Pepe, L. Storme and F. Vanhove, Theorems of Erdős-Ko-Rado type in polar spaces, *J. Combin. Theory Ser. A* **118** (2011), 1291–1312.

[2] S. Hobart, Bounds on subsets of coherent configurations, *Michigan Math. J.* **58** (2009), 231–239.

[3] S. Hobart and J. Williford, Tightness in subset bounds for coherent configurations, *J. Algebraic Combin.* (DOI) 10.1007/s10801-013-0459-4

Margarida Mitjana (Polytechnic University of Catalonia, Spain)
The Inverse Matrix of some Circulant and Symmetric Matrices

In this work, we consider circulant matrices of type $\mathsf{A} = \mathrm{Circ}(a, -b, -c, \dots, -c, b)$. This type of matrices raise, among others, when dealing for example, with the problem of computing the Green function of some networks obtained by the addition of new vertices to a previously known one. We give a necessary and sufficient condition for its invertibility. Moreover, as it is known, their inverse is a circulant matrix and we explicitely give a closed formula for the expression of the coefficients.

This is joint work with A. Carmona, A.M. Encinas, S. Gago and M.J. Jiménez.

Sonia Pérez (Polytechnic University of Catalonia, Spain)
A family of Abelian Cayley digraphs with asymptotically large order

In this talk we will deal with the degree-diameter problem of Cayley digraphs of Abelian groups. These digraphs can be constructed using a generalization to $\mathbb{Z}^n$ of the concept of congruence in $\mathbb{Z}$. We will use this approach to present an infinite family of such digraphs, which, for every fixed value of the degree, have asymptotically large number of vertices as the diameter increases.

Let $\mathrm{NA}_{d,k}$ (respectively, $\mathrm{NC}_{d,k}$) be the maximum number of vertices that a Cayley digraph of an Abelian group (respectively, of a cyclic group), with degree d and diameter k, can have. Wong and Coppersmith [4] proved that, for fixed degree d and large diameter k,

$$\left(\frac{k}{d}\right)^d + O(k^{d-1}) \le \mathrm{NC}_{d,k} \le \frac{k^d}{d!} + O(k^{d-1}). \tag{1}$$

The exact value of $\mathrm{NC}_{d,k}$ is only known for the case of degree $d = 2$ and several authors have given different dense families of Cayley digraphs of cyclic groups for $d = 3$. However, up to now no explicit proposal was known for the general case.

In the case of Cayley digraphs of Abelian groups, the bounds in (1) also apply for $\mathrm{NA}_{d,k}$. Nevertheless, the best upper bound known for the order of Cayley digraphs of Abelian groups is the one of Doughety and Faber in [1], where they proved the following nonconstructive result: There is a positive constant c (not depending on d or k), such that, for any fixed $d \ge 2$ and any k, there exist Cayley digraphs of Abelian groups on d generators having diameter at most k and number of vertices $N_{d,k}$ satisfying:

$$\mathrm{NA}_{d,k} \ge \frac{c}{d(\ln d)^{1+\log_2 e}} \frac{k^d}{d!} + O(k^{d-1}).$$

In our talk, we will show how to construct an infinite family of Cayley digraphs of Abelian groups whose order is asymptotically large as the diameter grows, using the following approach developed by Aguiló, Esqué and Fiol [2, 3]: Every Cayley digraph G from an Abelian group Γ is fully characterized by an integral $n \times n$ matrix M such that $\Gamma = \mathbb{Z}^n / M\mathbb{Z}^n$ (the so-called 'group of integral n-vectors modulo M'). Then, in such a representation, the digraph G is isomorphic to $\mathrm{Cay}(\mathbb{Z}^n/M\mathbb{Z}^n, A)$, where A is the set of unitary coordinate vectors $\boldsymbol{e}_i$, $i = 1, \ldots, n$.

This is joint work with F. Aguiló and M.A. Fiol.

References

[1] R. Dougherty and V. Faber, The degree-diameter problem for several varietes of Cayley graphs I: The Abelian case, *SIAM J. Discrete Math.* **17** (2004), no. 3, 478–519.

[2] P. Esqué, F. Aguiló, M.A. Fiol, Double commutative-step digraphs with minimum diameters, *Discrete Math.* **114** (1993) 147–157.

[3] M.A. Fiol, On congruences in $\mathbb{Z}^n$ and the dimension of a multidimensional circulant, *Discrete Math.* **141** (1995) 123–134.

[4] C.K. Wong, D. Coppersmith, A combinatorial problem related to multimodule memory organitzations, *J. Assoc. Comput. Machin.* **21** (1974) 392–402.

Bernardo Rodrigues (University of KwaZulu-Natal, South Africa)
Codes from incidence matrices of graphs

We examine the p-ary codes, for any prime p, from $|V| \times |E|$ incidence matrices of connected graphs $\Gamma = (V, E)$. We show that certain properties of the codes can be directly derived from properties of the graphs. For

example in the binary case the minimum distance of the code and the dual code equal the edge-connectivity of Γ (defined as the minimum number of edges whose removal renders Γ disconnected) and the girth(defined as the length of a shortest cycle) of Γ, respectively. In particular, we show that the binary code of a k-regular vertex transitive graph has minimum weight k and under further conditions the words of weight k are the rows of the incidence matrix. For the non-binary case we show that if Γ is a connected bipartite graph, a similar result holds under the same conditions. We examine also the implications for the binary codes from adjacency matrices of line graphs. Finally we show that the codes of some of these classes of graphs can be used for permutation decoding for full error correction with any information set.
This is joint work with Peter Dankelmann and Jennifer Key.

Participants

Invited speakers:
Aart Blokhuis (TUe Eindhoven, The Netherlands), Sebastian Cioabă (University of Delaware, USA), Miquel Àngel Fiol (Polytechnic University of Catalonia, Spain), Monique Laurent (CWI & Tilburg University), Oriol Serra (Polytechnic University of Catalonia, Spain).

Registered participants:
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